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ON THE ISSUE OF APPLYING THE PURE STRATEGIES SELECTION TACTICS IN THE MATRIX 2×2-GAME

There are stated the foundations of the concept of the pure strategies selection tactics for the matrix 2×2-game. Also there are considered some features of application of the pure strategies selection tactics with examples of the real 2×2-game, when the game plays quantity is finite.

Preamble and the problem wording

Matrix games are the simplest mathematical model of real conflict-controlled systems and processes. Having the efficient methods for determining the matrix game solution, there is deficiency of methods for realizing this solution in action, when the matrix game is not solved in pure strategies, but solved in mixed strategies. Even in 2×2-games with the empty set of saddle points in pure strategies there stands the problem of how to practice the two probabilities in the corresponding optimal mixed strategy

$$\hat{\mathbf{P}} = \left[\hat{p}_1 \quad \hat{p}_2\right] = \left[\hat{p}_1 \quad 1-\hat{p}_1\right] = \left[\hat{p} \quad 1-\hat{p}\right] \in \mathscr{P}_{\text{opt}} \subset \left\{\left[p_1 \quad p_2\right] \in \mathbb{R}^2 : p_k \in [0;1]\ \forall\ k=\overline{1,2},\ p_1+p_2=1\right\} \tag{1}$$

of the first player, and in the optimal mixed strategy

$$\breve{\mathbf{Q}} = \left[\breve{q}_1 \quad \breve{q}_2\right] = \left[\breve{q}_1 \quad 1-\breve{q}_1\right] = \left[\breve{q} \quad 1-\breve{q}\right] \in \mathscr{Q}_{\text{opt}} \subset \left\{\left[q_1 \quad q_2\right] \in \mathbb{R}^2 : q_l \in [0;1]\ \forall\ l=\overline{1,2},\ q_1+q_2=1\right\} \tag{2}$$

of the second player. Some angles of this problem are characterized in the paper [1], where the rationalized consecution or the tactics of the pure strategies selection is considered. The conclusion of this paper is that for realizing the optimal mixed strategies each of the players should not use a method of nonstochastic selection of the pure strategies. At the same time each of the players may practice its optimal mixed strategies (1) and (2), modeling the pure strategies selection with applying the independent variate, which is uniformly distributed on the semisegment $[0;1)$.

There in the paper [2] has been introduced the concept of the pure strategies selection tactics as a theoretic groundwork for investigating diverse ways of the optimal mixed strategies realization in the matrix 2×2-game with the finite game plays quantity. There have been deduced the formulas for finding the normed averaged by some realizations quantity mean payoffs of the first player for the fixed number of the game plays. There has been accomplished the conversion to the new matrix 2×2-game with the payoff matrix, which elements are the found normed averaged payoffs. With the solution of the presented new game in the pure strategies, if existing, it is possible to determine the optimal behavior of the players in the initial game with the finite game plays quantity. Now the goal is to view the application of the concept of the pure strategies selection tactics in real 2×2-games with the empty set of saddle points in pure strategies.

Application of the concept of the pure strategies selection tactics in real 2×2-games

For further, the matrix of the game will be assigned as

$$\mathbf{W} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \tag{3}$$

by

$$h_{kl} \in \mathbb{R} \quad \forall\ k=\overline{1,2},\ l=\overline{1,2}, \tag{4}$$

and the inequality

$$\max\left\{\min\left\{\{h_{11}, h_{12}\}, \{h_{21}, h_{22}\}\right\}\right\} < \min\left\{\max\left\{\{h_{11}, h_{21}\}, \{h_{12}, h_{22}\}\right\}\right\}, \tag{5}$$

that means that the set of saddle points in pure strategies is empty. Under the paper [1] conception, the first player raffles the uniformly distributed on the semisegment $[0;1)$ variate X and calculates the number

$$p_0 = \frac{\operatorname{sign}(\hat{p} - x) + 1}{2} \cdot \left|\operatorname{sign}(\hat{p} - x)\right| \tag{6}$$

by the value x of this variate. Clearly, that $p_0 \in \{0, 1\}$ and if $p_0 = 1$ then the first player should select the own first pure strategy s_{11}. By $p_0 = 0$ it selects the own second pure strategy s_{12}. Analogously the second player raffles the uniformly distributed on the semisegment $[0;1)$ variate Y and calculates the number

$$q_0 = \frac{\operatorname{sign}(\breve{q} - y) + 1}{2} \cdot \left|\operatorname{sign}(\breve{q} - y)\right| \tag{7}$$

by the value y of this variate. Here again $q_0 \in \{0, 1\}$ and by $q_0 = 1$ the second player should select the own first pure strategy s_{21}. If $q_0 = 0$ then it selects the own second pure strategy s_{22}. This mutual raffling and selecting the pure strategies by the values (6) and (7) allows to get the convergence between the mathematical expectation of the first player payoff and the game value

$$V_{\text{opt}} = \hat{\mathbf{P}}\mathbf{W}\breve{\mathbf{Q}}^{\mathrm{T}} = \begin{bmatrix} \hat{p} & 1-\hat{p} \end{bmatrix} \cdot \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \cdot \begin{bmatrix} \breve{q} \\ 1-\breve{q} \end{bmatrix}. \tag{8}$$

Developing the stated, under the paper [2] conception both the players should represent the probabilities $\hat{p}$ and $\breve{q}$ in the formulas (1) and (2) as the two fractions

$$\hat{p} = \frac{a}{d} \tag{9}$$

and

$$\breve{q} = \frac{b}{d} \tag{10}$$

with the identical least denominators, where

$$a \in \mathbb{N},\ b \in \mathbb{N},\ d \in \mathbb{N},\ a < d,\ b < d. \tag{11}$$

Then the two pure strategies selection tactics of the first player may be formed as the d-dimensional binary vector

$$\Theta_1 = \begin{bmatrix} \theta_{11} & \theta_{12} & \ldots & \theta_{1,d-1} & \theta_{1d} \end{bmatrix} \in \left\{ \Theta_1 \in \mathbb{R}^d : \theta_{1r} \in \{0, 1\},\ \sum_{r=1}^{d} \theta_{1r} = a \right\}. \tag{12}$$

Obviously, that the pure strategies selection tactics (12), for further being abbreviated to PSST, is a single PSST from the set of $C_d^a = \frac{d!}{a!(d-a)!}$ PSST of the first player. A single PSST from the set of $C_d^b = \frac{d!}{b!(d-b)!}$ PSST of the second player is the d-dimensional binary vector

$$\Theta_2 = \begin{bmatrix} \theta_{21} & \theta_{22} & \ldots & \theta_{2,d-1} & \theta_{2d} \end{bmatrix} \in \left\{ \Theta_2 \in \mathbb{R}^d : \theta_{2r} \in \{0, 1\},\ \sum_{r=1}^{d} \theta_{2r} = b \right\}. \tag{13}$$

Consequently, in the considered game both players may apply totally the quantity

$$C_d^a \cdot C_d^b = \frac{d!}{a!(d-a)!} \cdot \frac{d!}{b!(d-b)!} = \frac{(d!)^2}{a!b!(d-a)!(d-b)!} \tag{14}$$

of pairs of different PSST. May the set

$$\Omega_1 = \left\{ \Theta_1(1),\quad \Theta_1(2),\quad \ldots,\quad \Theta_1\left(C_d^a - 1\right),\quad \Theta_1\left(C_d^a\right) \right\} \tag{15}$$

of all C_d^a PSST of the first player be sorted by the following way. The elements of the set (15) correspond to the binary d -digit numbers, which are sorted in descending order from the number

$$(1\ \ 1\ \ \ldots\ \ 1\ \ 0\ \ 0\ \ \ldots\ \ 0)_2 = \left(\theta_{11}(1)\ \ \theta_{12}(1)\ \ \ldots\ \ \theta_{1a}(1)\ \ \theta_{1,a+1}(1)\ \ \theta_{1,a+2}(1)\ \ \ldots\ \ \theta_{1d}(1)\right)_2 \tag{16}$$

with $\theta_{1i}(1) = 1\ \ \forall\ i = \overline{1, a}$, $\theta_{1j}(1) = 0\ \ \forall\ j = \overline{a+1, d}$, down to the number

$$\begin{gathered}(0\ \ 0\ \ \ldots\ \ 0\ \ 1\ \ 1\ \ \ldots\ \ 1)_2 = \\ = \left(\theta_{11}\left(C_d^a\right)\ \ \theta_{12}\left(C_d^a\right)\ \ \ldots\ \ \theta_{1,d-a}\left(C_d^a\right)\ \ \theta_{1,d-a+1}\left(C_d^a\right)\ \ \theta_{1,d-a+2}\left(C_d^a\right)\ \ \ldots\ \ \theta_{1d}\left(C_d^a\right)\right)_2\end{gathered} \tag{17}$$

with $\theta_{1,\,j-a}\left(C_d^a\right) = 0\ \ \forall\ j = \overline{a+1, d}$, $\theta_{1,\,i+d-a}\left(C_d^a\right) = 1\ \ \forall\ i = \overline{1, a}$. Thus, the first element $\Theta_1(1)$ of the set (15) corresponds to the binary number (16), and the last element $\Theta_1\left(C_d^a\right)$ of this set corresponds to the binary number (17). Analogously the set

$$\Omega_2 = \left\{ \Theta_2(1),\quad \Theta_2(2),\quad \ldots,\quad \Theta_2\left(C_d^b - 1\right),\quad \Theta_2\left(C_d^b\right) \right\} \tag{18}$$

of all C_d^b PSST of the second player is sorted as the elements of the set (18) correspond to the binary d -digit numbers, which are sorted in descending order from the number

$$(1\ \ 1\ \ \ldots\ \ 1\ \ 0\ \ 0\ \ \ldots\ \ 0)_2 = \left(\theta_{21}(1)\ \ \theta_{22}(1)\ \ \ldots\ \ \theta_{2b}(1)\ \ \theta_{2,b+1}(1)\ \ \theta_{2,b+2}(1)\ \ \ldots\ \ \theta_{2d}(1)\right)_2 \tag{19}$$

with $\theta_{2i}(1) = 1\ \ \forall\ i = \overline{1, b}$, $\theta_{2j}(1) = 0\ \ \forall\ j = \overline{b+1, d}$, down to the number

$$\begin{gathered}(0\ \ 0\ \ \ldots\ \ 0\ \ 1\ \ 1\ \ \ldots\ \ 1)_2 = \\ = \left(\theta_{21}\left(C_d^b\right)\ \ \theta_{22}\left(C_d^b\right)\ \ \ldots\ \ \theta_{2,d-b}\left(C_d^b\right)\ \ \theta_{2,d-b+1}\left(C_d^b\right)\ \ \theta_{2,d-b+2}\left(C_d^b\right)\ \ \ldots\ \ \theta_{2d}\left(C_d^b\right)\right)_2\end{gathered} \tag{20}$$

with $\theta_{2,\,j-b}\left(C_d^b\right) = 0\ \ \forall\ j = \overline{b+1, d}$, $\theta_{2,\,i+d-b}\left(C_d^b\right) = 1\ \ \forall\ i = \overline{1, b}$. And here the first element $\Theta_2(1)$ of the set (18) corresponds to the binary number (19), and the last element $\Theta_2\left(C_d^b\right)$ of this set corresponds to the binary number (20).

Providing that in a series from $G \in \mathbb{N}$ plays [2] the players apply independently a pair of PSST $\left\{\Theta_1(c_1), \Theta_2(c_2)\right\}$ with $c_1 \in \left\{\overline{1, C_d^a}\right\}$ and $c_2 \in \left\{\overline{1, C_d^b}\right\}$, in the g -th play, $g = \overline{1, G}$, the first player selects its pure strategy by the pattern in the 2-element vector

$$\mathbf{T}_1(c_1, g) \in \left\{[1\ \ \ 0], [0\ \ \ 1]\right\}, \tag{21}$$

and meanwhile the second player selects its pure strategy by the pattern in the 2-element vector

$$\mathbf{T}_2(c_2, g) \in \left\{[1\ \ \ 0], [0\ \ \ 1]\right\}. \tag{22}$$

If $g \leqslant d$ then for $\theta_{1g}(c_1) = 0$ the vector (21) is

$$\mathbf{T}_1(c_1, g) = [0\ \ \ 1], \tag{23}$$

and for $\theta_{2g}(c_2) = 0$ the vector (22) is

$$\mathbf{T}_2(c_2, g) = [0\ \ \ 1]; \tag{24}$$

for $\theta_{1g}(c_1)=1$ the vector (21) is

$$\mathbf{T}_1(c_1,\, g)=[1 \quad 0], \tag{25}$$

and for $\theta_{2g}(c_2)=1$ the vector (22) is

$$\mathbf{T}_2(c_2,\, g)=[1 \quad 0]. \tag{26}$$

if $g>d$ then both the players apply their PSST cyclically, that is by $g=m+nd$ and $m<d$ with $n\in\mathbb{N}$ there will be

$$\mathbf{T}_1(c_1,\, g)=\mathbf{T}_1(c_1,\, m), \tag{27}$$

$$\mathbf{T}_2(c_2,\, g)=\mathbf{T}_2(c_2,\, m). \tag{28}$$

Due to the paper [2] results, the players for optimal selecting their pure strategies may either apply the values (6) and (7), or apply their cyclical PSST (21) — (28). Incidentally, the first mentioned way of the selection may be called the stochastic PSST. For evaluating the expected payoff of the first player in G plays, there is the necessity to repeat these G plays for R times, where $R\in\mathbb{N}\setminus\{1\}$ generally. If both the players select pure strategies applying the values (6) and (7), then may $w_{11}(g,\, r;\, G,\, R)$ be a first player payoff in the g -th play by the r -th repeat of the G plays series, where $r=\overline{1,\, R}$. And then the normed averaged first player payoff is

$$v_{11}(G,\, R)=\frac{V_{11}(G,\, R)-V_{\text{opt}}}{|V_{\text{opt}}|}=\frac{\frac{1}{RG}\sum_{r=1}^{R}\sum_{g=1}^{G}w_{11}(g,\, r;\, G,\, R)-V_{\text{opt}}}{|V_{\text{opt}}|} \tag{29}$$

by

$$w_{11}(g,\, r;\, G,\, R)=[p_0 \quad 1-p_0]\cdot\mathbf{W}\cdot[q_0 \quad 1-q_0]^{\mathrm{T}}. \tag{30}$$

If the first player applies the value (6), and the second player applies the PSST $\Theta_2(c_2)$, then the normed averaged first player payoff is

$$v_{10}(c_2;\, G,\, R)=\frac{V_{10}(c_2;\, G,\, R)-V_{\text{opt}}}{|V_{\text{opt}}|}=\frac{\frac{1}{RG}\sum_{r=1}^{R}\sum_{g=1}^{G}w_{10}(g,\, r;\, c_2;\, G,\, R)-V_{\text{opt}}}{|V_{\text{opt}}|} \tag{31}$$

by a first player payoff in the g -th play by the r -th repeat of the G plays series with applying the PSST $\Theta_2(c_2)$

$$w_{10}(g,\, r;\, c_2;\, G,\, R)=[p_0 \quad 1-p_0]\cdot\mathbf{W}\cdot[\mathbf{T}_2(c_2,\, g)]^{\mathrm{T}}, \tag{32}$$

where $r=\overline{1,\, R}$. For the third case, if the first player applies the PSST $\Theta_1(c_1)$, and the second player applies the value (7), then the normed averaged first player payoff is

$$v_{01}(c_1;\, G,\, R)=\frac{V_{01}(c_1;\, G,\, R)-V_{\text{opt}}}{|V_{\text{opt}}|}=\frac{\frac{1}{RG}\sum_{r=1}^{R}\sum_{g=1}^{G}w_{01}(g,\, r;\, c_1;\, G,\, R)-V_{\text{opt}}}{|V_{\text{opt}}|} \tag{33}$$

by a first player payoff in the g -th play by the r -th repeat of the G plays series with applying the PSST $\Theta_1(c_1)$

$$w_{01}(g,\, r;\, c_1;\, G,\, R)=\mathbf{T}_1(c_1,\, g)\cdot\mathbf{W}\cdot[q_0 \quad 1-q_0]^{\mathrm{T}}, \tag{34}$$

where $r = \overline{1, R}$. At last, if both the players apply their PSST $\Theta_1(c_1)$ and $\Theta_2(c_2)$ as the independent elements in the pair $\{\Theta_1(c_1), \Theta_2(c_2)\}$, then the normed averaged first player payoff is

$$v_{00}(c_1, c_2; G, R) = \frac{V_{00}(c_1, c_2; G, R) - V_{\text{opt}}}{|V_{\text{opt}}|} = \frac{\frac{1}{RG}\sum_{r=1}^{R}\sum_{g=1}^{G} w_{00}(g, r; c_1, c_2; G, R) - V_{\text{opt}}}{|V_{\text{opt}}|} \tag{35}$$

by a first player payoff in the g-th play by the r-th repeat of the G plays series

$$w_{00}(g, r; c_1, c_2; G, R) = \mathbf{T}_1(c_1, g) \cdot \mathbf{W} \cdot \left[\mathbf{T}_2(c_2, g)\right]^{\mathrm{T}} \tag{36}$$

with applying the PSST pair $\{\Theta_1(c_1), \Theta_2(c_2)\}$.

Consequently, each of the players has two ways of making the considered game, when it applies the relevant values (6) and (7), or applies a PSST. Then there is a new 2×2-game with the matrix

$$\mathbf{V}(c_1, c_2; G, R) = \begin{bmatrix} v_{11}(G, R) & v_{10}(c_2; G, R) \\ v_{01}(c_1; G, R) & v_{00}(c_1, c_2; G, R) \end{bmatrix} \tag{37}$$

by the clear limit

$$\lim_{G\to\infty} v_{11}(G, R) = \lim_{G\to\infty} \frac{V_{11}(G, R) - V_{\text{opt}}}{|V_{\text{opt}}|} = \lim_{G\to\infty} \frac{\frac{1}{RG}\sum_{r=1}^{R}\sum_{g=1}^{G} w_{11}(g, r; G, R) - V_{\text{opt}}}{|V_{\text{opt}}|} =$$
$$= \lim_{R\to\infty} v_{11}(G, R) = \lim_{R\to\infty} \frac{V_{11}(G, R) - V_{\text{opt}}}{|V_{\text{opt}}|} = \lim_{R\to\infty} \frac{\frac{1}{RG}\sum_{r=1}^{R}\sum_{g=1}^{G} w_{11}(g, r; G, R) - V_{\text{opt}}}{|V_{\text{opt}}|} =$$
$$= \frac{\begin{bmatrix} \widehat{p} & 1-\widehat{p} \end{bmatrix} \cdot \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \cdot \begin{bmatrix} \breve{q} \\ 1-\breve{q} \end{bmatrix} - V_{\text{opt}}}{|V_{\text{opt}}|} = \frac{V_{\text{opt}} - V_{\text{opt}}}{|V_{\text{opt}}|} = 0\,. \tag{38}$$

For applying the concept of the PSST in real 2×2-games there in MATLAB 7.0.1 had been programmed the module `psst2by2` (figure 1) for computing the matrix (37) and further solving the obtained 2×2-game. This module works in the purport, that for any 2×2-game with the empty set of saddle points in pure strategies there in MATLAB 7.0.1 Command Window are returned the solution (1), (2), (8), the number d as the PSST period, randomly formed PSST for each player, the normed averaged first player payoffs (29), (31), (33), (35) for the input initially G and R, and the solution of the 2×2-game with the matrix (37). It should be marked, that instead of G there is possibility to input the range of the changing G by entering two natural numbers as `MIN_TotalPlaysNumber` and `MAX_TotalPlaysNumber`, where the first number is not greater than the second. Entering two equal numbers means that the investigation will be carrying for the single value of the game plays G. As for the rest two arguments of the module `psst2by2`, then `PayoffMatrix` is the matrix (3), and `Realizations` is the number R.

As an example, the solution of the 2×2-game with the matrix

$$\mathbf{W} = \begin{bmatrix} 8 & 2 \\ 1 & 14 \end{bmatrix} \tag{39}$$

is (figure 2, figure 3)

$$\widehat{\mathbf{P}} = \begin{bmatrix} \widehat{p}_1 & \widehat{p}_2 \end{bmatrix} = \begin{bmatrix} \widehat{p}_1 & 1-\widehat{p}_1 \end{bmatrix} = \begin{bmatrix} \widehat{p} & 1-\widehat{p} \end{bmatrix} = \begin{bmatrix} \frac{13}{19} & \frac{6}{19} \end{bmatrix}, \tag{40}$$

$$\breve{\mathbf{Q}} = \begin{bmatrix} \breve{q}_1 & \breve{q}_2 \end{bmatrix} = \begin{bmatrix} \breve{q}_1 & 1-\breve{q}_1 \end{bmatrix} = \begin{bmatrix} \breve{q} & 1-\breve{q} \end{bmatrix} = \begin{bmatrix} \frac{12}{19} & \frac{7}{19} \end{bmatrix}, \tag{41}$$

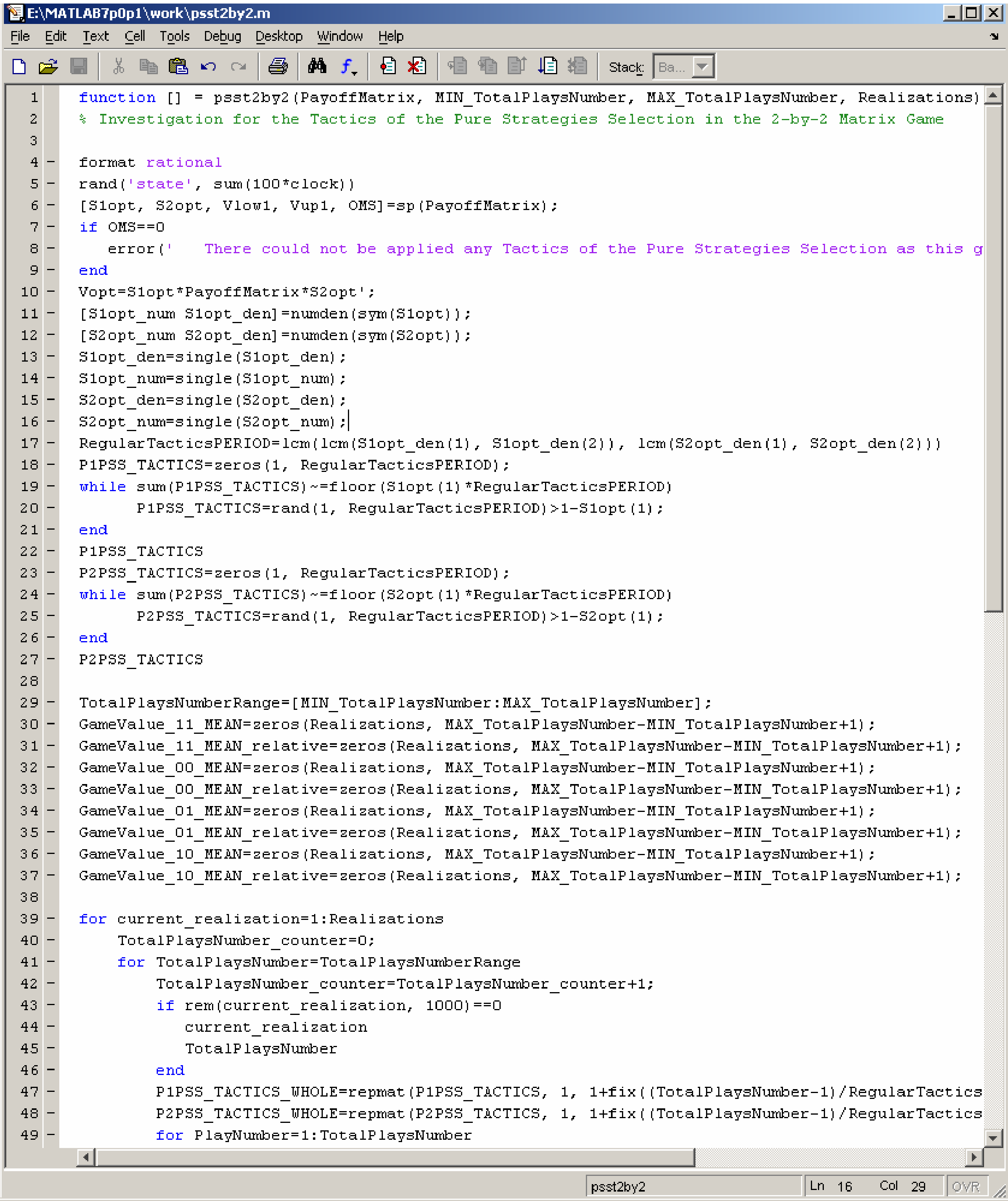
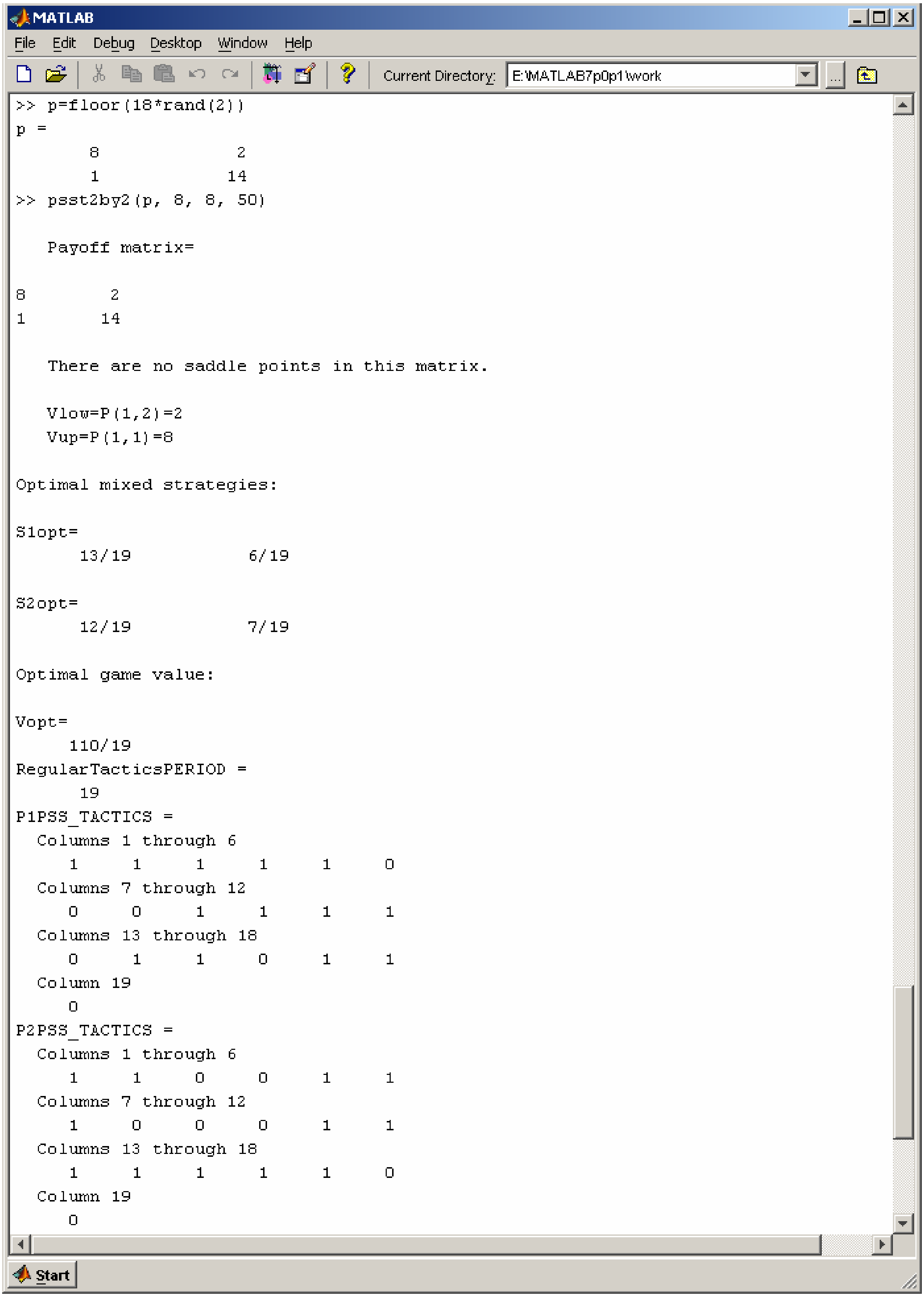


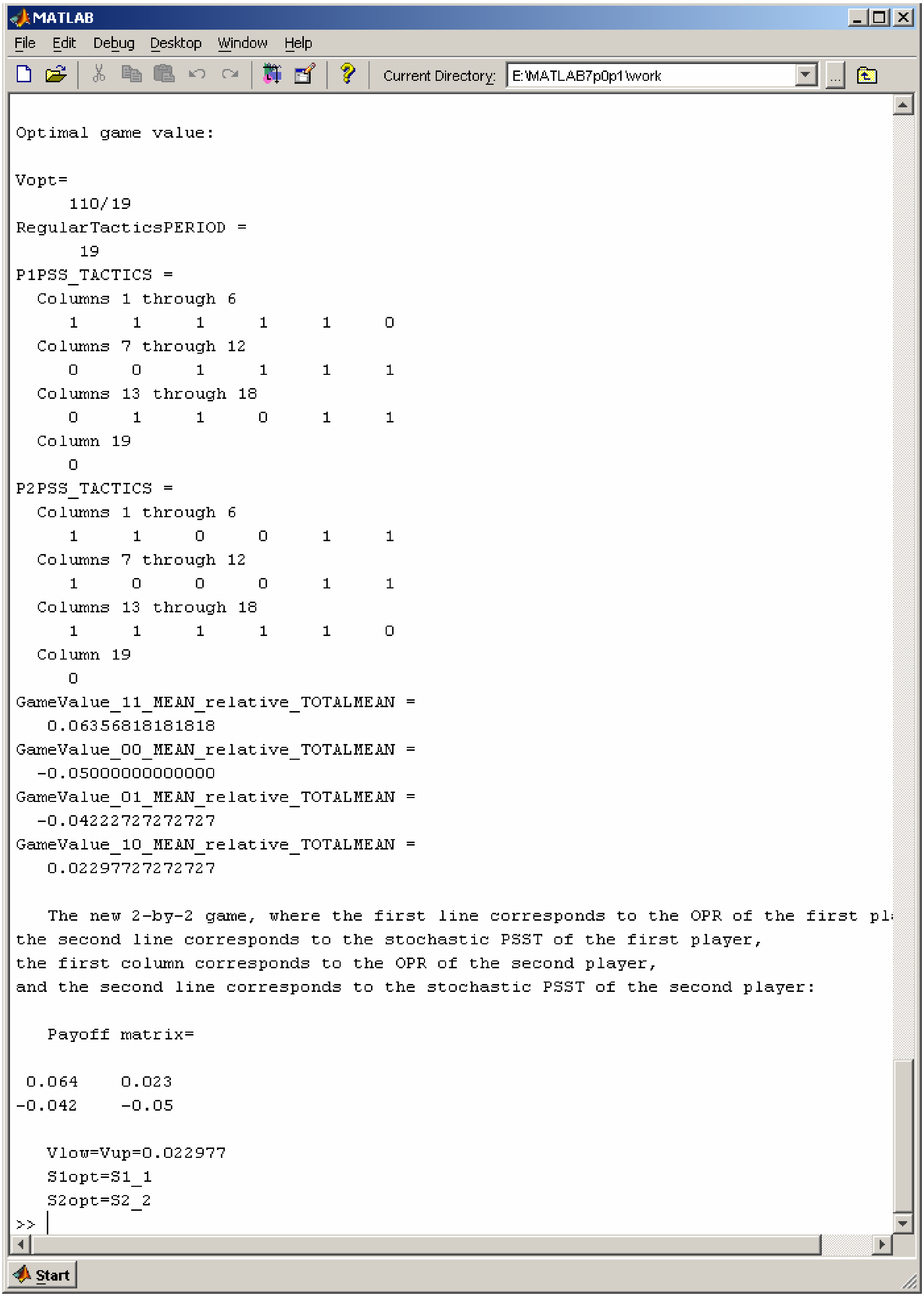
Figure 1. The program module `psst2by2` window in MATLAB 7.0.1 M-file Editor

$$V_{\text{opt}} = \hat{\mathbf{P}}\mathbf{W}\breve{\mathbf{Q}}^{\mathrm{T}} = \begin{bmatrix} \hat{p} & 1-\hat{p} \end{bmatrix} \cdot \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \cdot \begin{bmatrix} \breve{q} \\ 1-\breve{q} \end{bmatrix} = \begin{bmatrix} \frac{13}{19} & \frac{6}{19} \end{bmatrix} \cdot \begin{bmatrix} 8 & 2 \\ 1 & 14 \end{bmatrix} \cdot \begin{bmatrix} \frac{12}{19} \\ \frac{7}{19} \end{bmatrix} = \frac{110}{19}. \tag{42}$$



```
>> p=floor(18*rand(2))
p =
       8              2
       1             14
>> psst2by2(p, 8, 8, 50)

   Payoff matrix=

8         2
1        14

   There are no saddle points in this matrix.

   Vlow=P(1,2)=2
   Vup=P(1,1)=8

Optimal mixed strategies:

S1opt=
      13/19           6/19

S2opt=
      12/19           7/19

Optimal game value:

Vopt=
     110/19
RegularTacticsPERIOD =
      19
P1PSS_TACTICS =
  Columns 1 through 6
     1     1     1     1     1     0
  Columns 7 through 12
     0     0     1     1     1     1
  Columns 13 through 18
     0     1     1     0     1     1
  Column 19
     0
P2PSS_TACTICS =
  Columns 1 through 6
     1     1     0     0     1     1
  Columns 7 through 12
     1     0     0     0     1     1
  Columns 13 through 18
     1     1     1     1     1     0
  Column 19
     0
```

Figure 2. The solution of the game with the randomly formed payoff matrix (39) in the accomplishment of the module `psst2by2` in MATLAB 7.0.1 Command Window for this matrix with $G = 8$ and $R = 50$



```
MATLAB
File Edit Debug Desktop Window Help
Current Directory: E:\MATLAB7p0p1\work

Optimal game value:

Vopt=
     110/19
RegularTacticsPERIOD =
      19
P1PSS_TACTICS =
  Columns 1 through 6
     1     1     1     1     1     0
  Columns 7 through 12
     0     0     1     1     1     1
  Columns 13 through 18
     0     1     1     0     1     1
  Column 19
     0
P2PSS_TACTICS =
  Columns 1 through 6
     1     1     0     0     1     1
  Columns 7 through 12
     1     0     0     0     1     1
  Columns 13 through 18
     1     1     1     1     1     0
  Column 19
     0
GameValue_11_MEAN_relative_TOTALMEAN =
   0.06356818181818
GameValue_00_MEAN_relative_TOTALMEAN =
  -0.05000000000000
GameValue_01_MEAN_relative_TOTALMEAN =
  -0.04222727272727
GameValue_10_MEAN_relative_TOTALMEAN =
   0.02297727272727

   The new 2-by-2 game, where the first line corresponds to the OPR of the first pl
the second line corresponds to the stochastic PSST of the first player,
the first column corresponds to the OPR of the second player,
and the second line corresponds to the stochastic PSST of the second player:

   Payoff matrix=

 0.064    0.023
-0.042    -0.05

   Vlow=Vup=0.022977
   S1opt=S1_1
   S2opt=S2_2
>>
Start
```

Figure 3. The final results of the accomplishment of the module `psst2by2` in MATLAB 7.0.1 Command Window for the randomly formed payoff matrix (39) with $G = 8$ and $R = 50$

It is seen from the figure 3, that the final results of the accomplishment of the module `psst2by2` give the saddle point for the matrix (37), that is for improving one's situation the first player may apply the value (6), while the second may apply its specified PSST. Nevertheless, if the 50 series by the 8 plays would end like this, the first player would have the final advantage, as the determined saddle point is positive. But clearly, that $v_{11}(G, R)$, $v_{10}(c_2; G, R)$ and $v_{01}(c_1; G, R)$ are the values of some variates, as they are determined through the variates p_0 and q_0. And the number $v_{00}(c_1, c_2; G, R)$ is not a variate as it depends on the specified PSST $\Theta_1(c_1)$ and $\Theta_2(c_2)$. So, the obtained result is random, though for the greater number of realizations R there may display some stable features. Some of them concern the fact that the players should never apply simultaneously their cyclical PSST, as this causes the disadvantageous normed averaged payoff for one of the players. On the figures 4 — 10, been print-screened after the accomplishment of the module `psst2by2` in MATLAB 7.0.1 Command Window for the randomly formed payoff matrix (39) with $G = 8$ and $R = 50000$, and on the figures 11 — 13, been print-screened after the accomplishment of the module `psst2by2` in MATLAB 7.0.1 Command Window for the randomly formed payoff matrix (39) with $G = 8$ and $R = 10^6$, the said fact is stated with the inequality

$$\left|v_{00}(c_1, c_2; G, R)\right| > \max\left\{\left|v_{11}(G, R)\right|, \left|v_{10}(c_2; G, R)\right|, \left|v_{01}(c_1; G, R)\right|\right\} \tag{43}$$

by some randomly specified PSST $\Theta_1(c_1)$ and $\Theta_2(c_2)$.

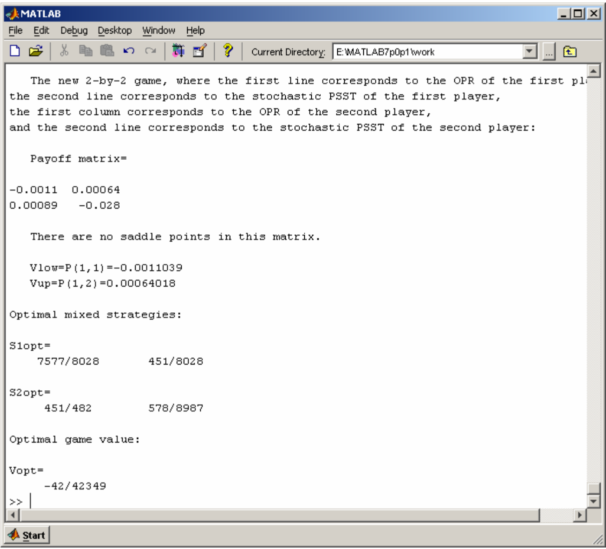


Figure 4

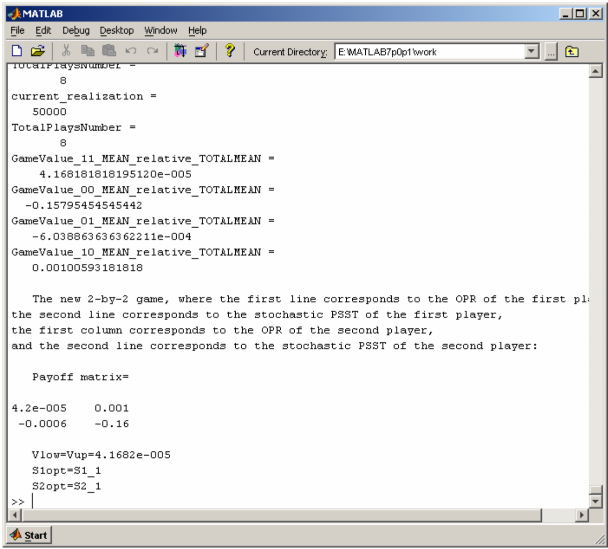


Figure 5

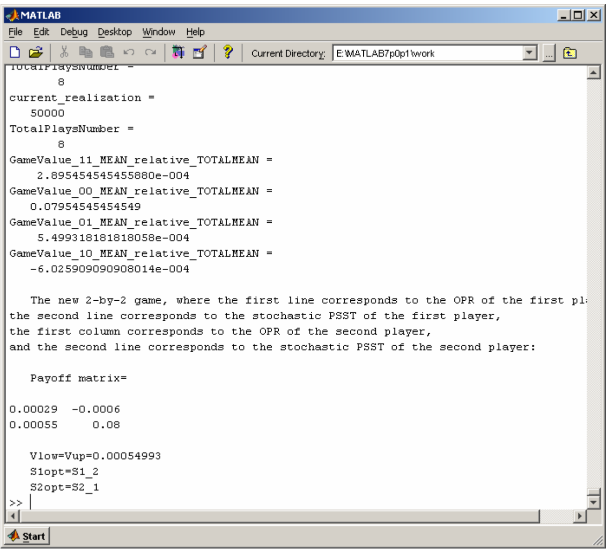


Figure 6

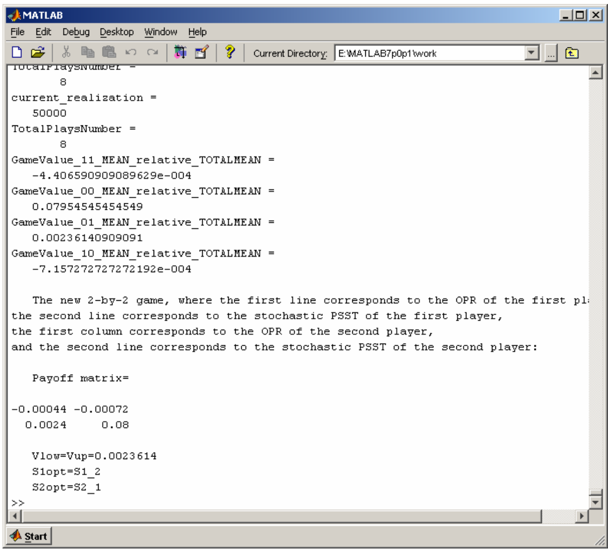


Figure 7

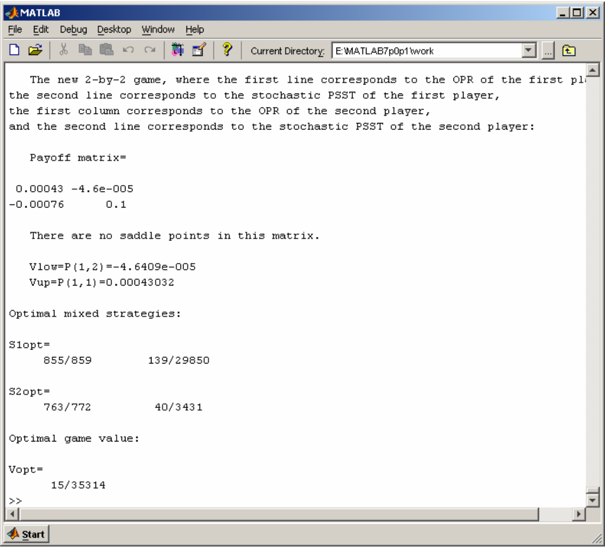


Figure 8

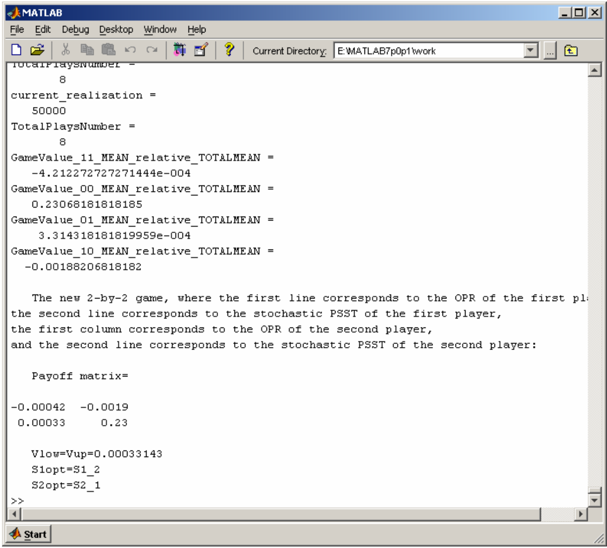


Figure 9

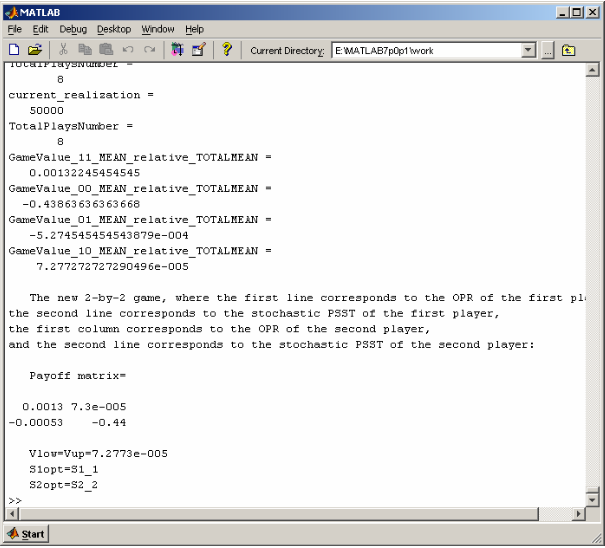


Figure 10

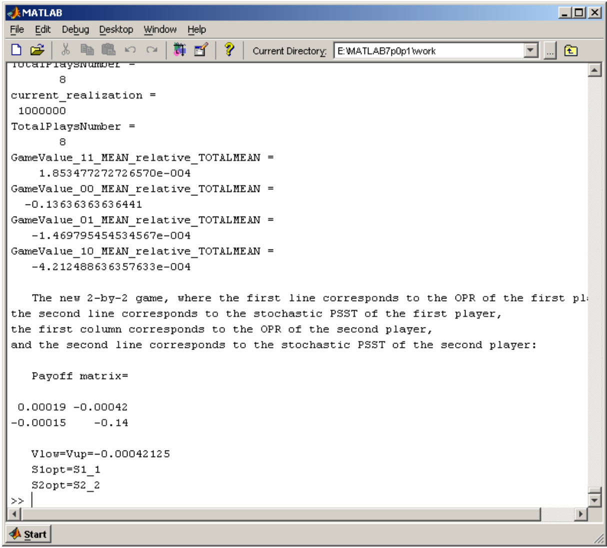


Figure 11

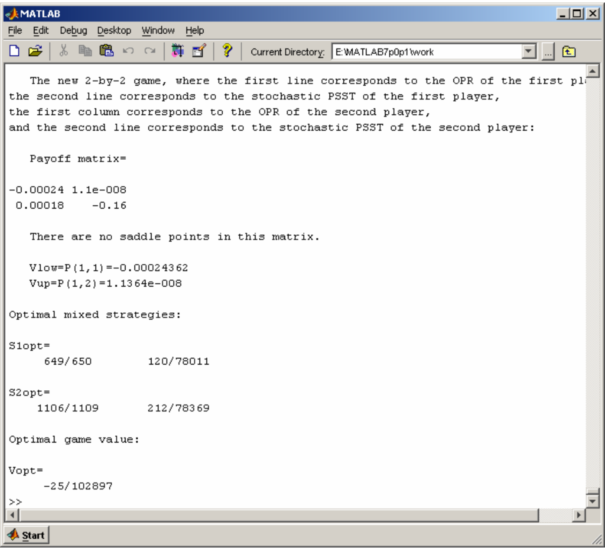


Figure 12

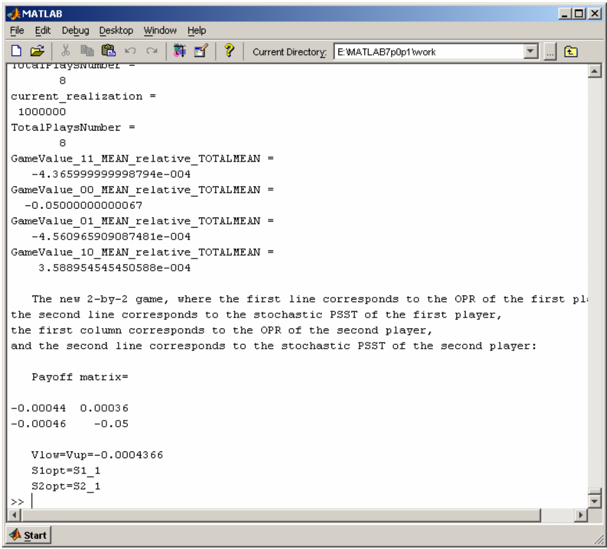


Figure 13

Therefore in realizing the solution (40) — (42) in action, one of the players should certainly apply the known methods of practicing the optimal mixed strategy [1, 2], which mainly based on the calculation of the values (6) and (7). What is more, that the player must be aware that if it applies a PSST then by the other player applying a PSST there is a risk of a great averaged loss for one of them. Such a risk is evinced clearly when the new 2×2-game (37) has the empty set of saddle points in pure strategies, and the players should apply the mixed strategies in this peculiar metagame. For print-screened instances, it is well seen from the figures 4 and 12, that when the first player risks with applying a PSST, then it may lose normally 0.028 or 0.16 in particular. The second player, as it is on the figure 8, with applying some of its PSST may lose up to 0.1. But the shown in the figures 4 — 13 results are random, as well as the PSST application by a player, seen from the figures 6, 7, 9 — 11. Though, for knowing in advance what PSST a player will apply, the other player may compute the value $v_{00}(c_1, c_2; G, R)$ and, actually, the matrix (37), and make a decision whether it applies own PSST or practices the optimal mixed strategy, based on the calculation of the values (6) and (7).

As an example, if in the game with the matrix

$$\mathbf{W} = \begin{bmatrix} 0 & 5 \\ 6 & -3 \end{bmatrix} \tag{44}$$

and the solution

$$\widehat{\mathbf{P}} = [\widehat{p}_1 \quad \widehat{p}_2] = [\widehat{p}_1 \quad 1 - \widehat{p}_1] = [\widehat{p} \quad 1 - \widehat{p}] = \begin{bmatrix} \frac{9}{14} & \frac{5}{14} \end{bmatrix}, \tag{45}$$

$$\breve{\mathbf{Q}} = [\breve{q}_1 \quad \breve{q}_2] = [\breve{q}_1 \quad 1 - \breve{q}_1] = [\breve{q} \quad 1 - \breve{q}] = \begin{bmatrix} \frac{4}{7} & \frac{3}{7} \end{bmatrix}, \tag{46}$$

$$V_{\mathrm{opt}} = \widehat{\mathbf{P}}\mathbf{W}\breve{\mathbf{Q}}^{\mathrm{T}} = [\widehat{p} \quad 1 - \widehat{p}] \cdot \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \cdot \begin{bmatrix} \breve{q} \\ 1 - \breve{q} \end{bmatrix} = \\ = \begin{bmatrix} \frac{9}{14} & \frac{5}{14} \end{bmatrix} \cdot \begin{bmatrix} 0 & 5 \\ 6 & -3 \end{bmatrix} \cdot \begin{bmatrix} \frac{4}{7} \\ \frac{3}{7} \end{bmatrix} = \frac{15}{7}, \tag{47}$$

the first player certainly knows that the second will apply the PSST

$$\Theta_2(1) = [\theta_{21}(1) \quad \theta_{22}(1) \quad \ldots \quad \theta_{2,13}(1) \quad \theta_{2,14}(1)] = \\ = [1 \; 1 \; 1 \; 1 \; 1 \; 1 \; 1 \; 1 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0] \in \mathbb{R}^{14} \tag{48}$$

for the planned preset number of plays $G = 10$, then the first player may compute, what exact of the $C_{14}^{9} = 2002$ PSST gives the greatest positive number $v_{00}(c_1, c_2; G, R)$. And of course, it is remarkable that here the result does not depend on the number of realizations R as the value $v_{00}(c_1, c_2; G, R)$ is not a variate.

If the first player applies the PSST

$$\Theta_1(1) = [\theta_{11}(1) \quad \theta_{12}(1) \quad \ldots \quad \theta_{1,13}(1) \quad \theta_{1,14}(1)] = \\ = [1 \; 1 \; 1 \; 1 \; 1 \; 1 \; 1 \; 1 \; 1 \; 0 \; 0 \; 0 \; 0 \; 0] \in \mathbb{R}^{14} \tag{49}$$

then it loses normally 0.91 (figure 14). If it applies the PSST

$$\Theta_1(u_1) = [\theta_{11}(u_1) \quad \theta_{12}(u_1) \quad \ldots \quad \theta_{1,13}(u_1) \quad \theta_{1,14}(u_1)] = \\ = [0 \; 0 \; 1 \; 1 \; 1 \; 0 \; 1 \; 0 \; 1 \; 1 \; 1 \; 0 \; 1 \; 1] \in \mathbb{R}^{14} \tag{50}$$

by $1 < u_1 < 2002$ without calculating the number u_1, then the first player gets the normed averaged payoff 0.59 (figure 15). Though, with a slight change of PSST into the form

$$\Theta_1(u_2) = [\theta_{11}(u_2) \quad \theta_{12}(u_2) \quad \ldots \quad \theta_{1,13}(u_2) \quad \theta_{1,14}(u_2)] = \\ = [1 \; 0 \; 1 \; 1 \; 1 \; 0 \; 1 \; 0 \; 1 \; 1 \; 1 \; 0 \; 1 \; 0] \in \mathbb{R}^{14} \tag{51}$$

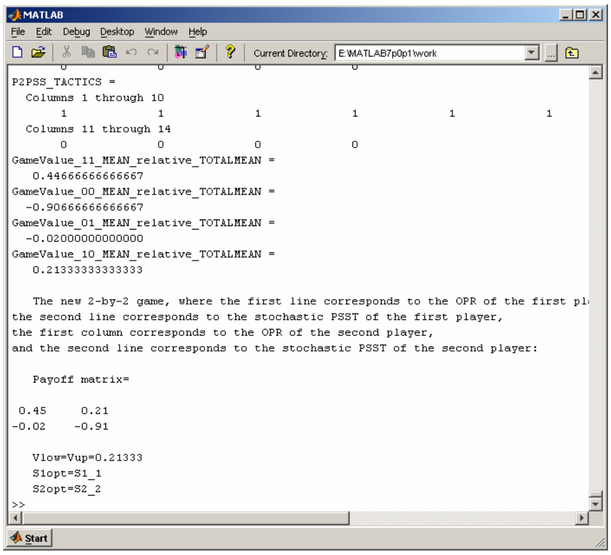


Figure 14

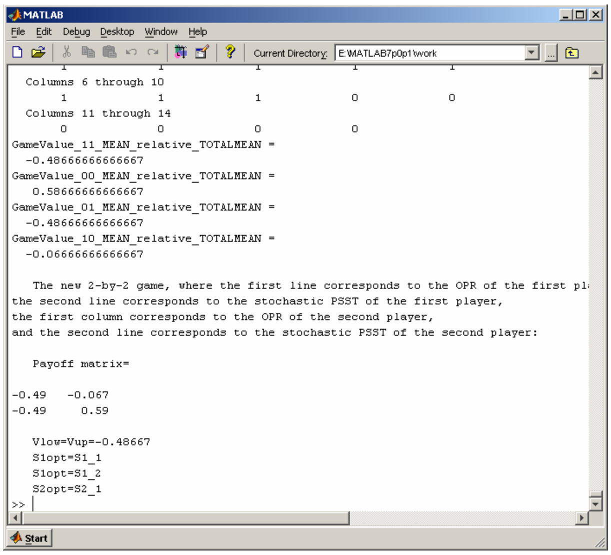


Figure 15

by $1 < u_2 < 2002$ without calculating the number $u_2 \neq u_1$, the first player reduces its normed averaged payoff down to 0.31 (figure 16). However, with applying the PSST

$$\Theta_1(2002) = \left[\theta_{11}(2002) \quad \theta_{12}(2002) \quad \dots \quad \theta_{1,13}(2002) \quad \theta_{1,14}(2002)\right] =$$
$$= \left[0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1\right] \in \mathbb{R}^{14} \tag{52}$$

the first player gets the normed averaged payoff 0.87 (figure 17), that is for every series of $G = 10$ plays of the game with the matrix (44) the first player will get the averaged payoff equal to 4 instead of the optimal game value (47).

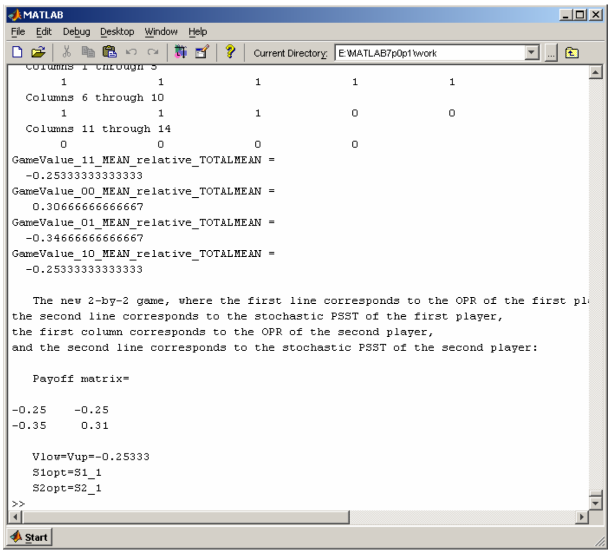


Figure 16

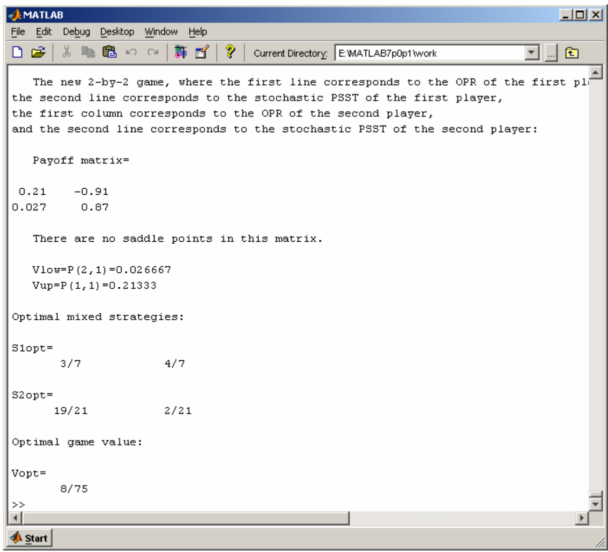


Figure 17

But as the second player digresses from applying the PSST (48), and starts applying the stochastically formed PSST, then the first player with PSST (52) may lose as exampled on the figures 18 and 19. In the case shown with the figure 18, the first player would lose the normed averaged payoff 0.63, but in the case shown with the figure 19, the first player would lose the greater normed averaged payoff, equal to 0.81. The similar occurrences are shown with the figures 20 and 21.

Nevertheless, in general, the most reliable way is to hold by the calculation of the values (6) and (7), that is directly corroborated with the figures 5 and 13, and, by implication, with the figures 4, 8 and 12, where the great optimal probabilities of holding by the calculation of the values (6) and (7) mean that this holding is almost the statistically defined event.

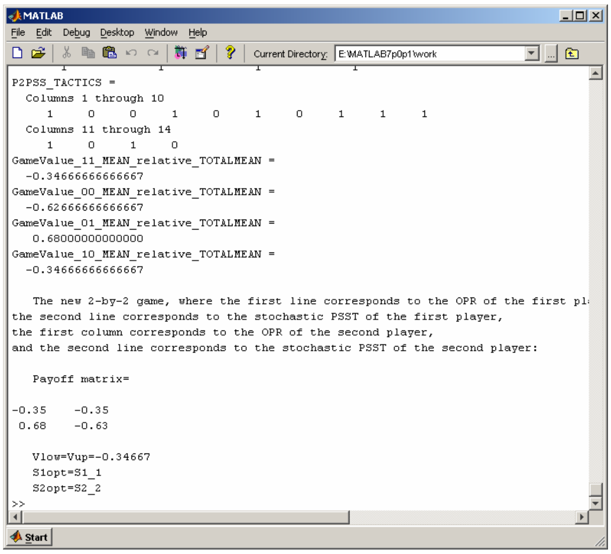


Figure 18

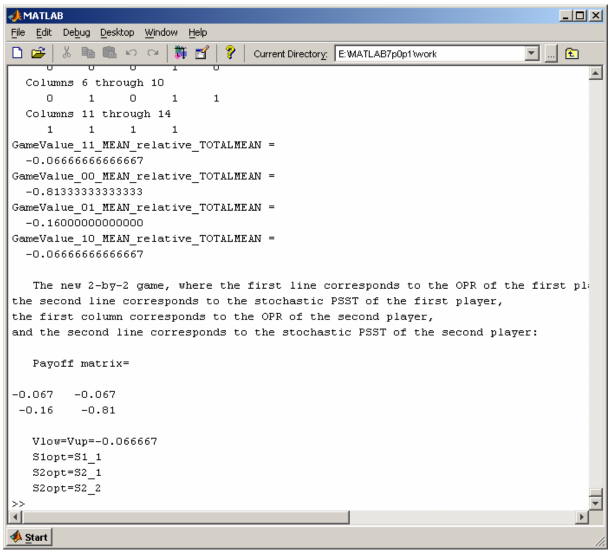


Figure 19

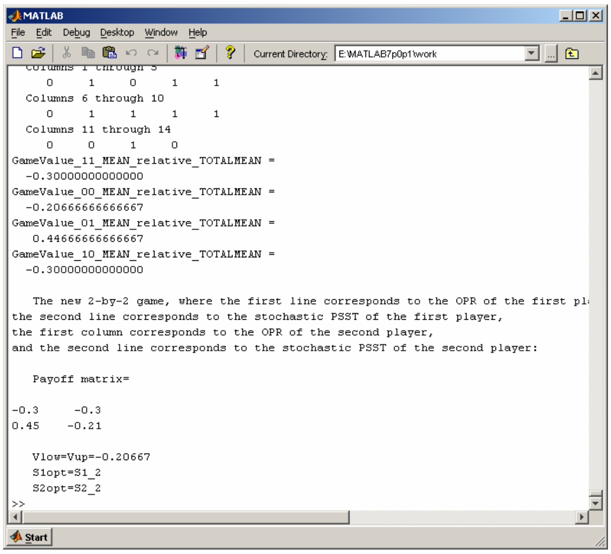


Figure 20

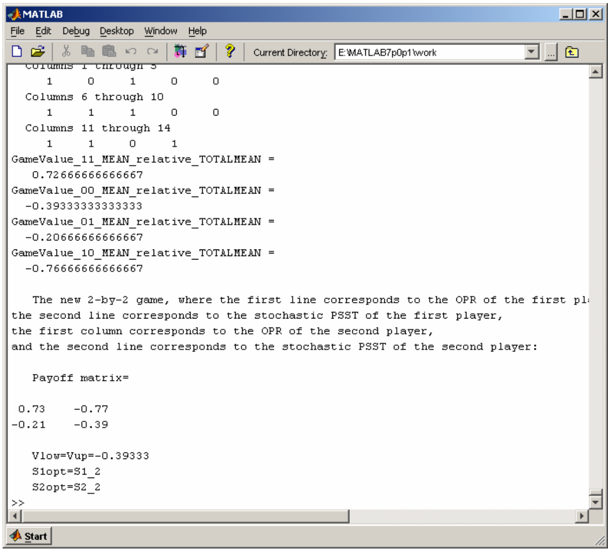


Figure 21

Conclusion paragraph

As the print-screened figures have demonstrated, for realizing the optimal mixed strategies in the 2×2-game each of the players should use a method of stochastic selection of the pure strategies. The conception of applying the pure strategies selection tactics in real 2×2-games with the empty set of saddle points in pure strategies is intended for those games, where one of the players had planned in advance to select its pure strategies by some schedule. This schedule may be called the PSST of that player. And then the other player, if knowing in advance or understanding partially that schedule, may compute some own PSST, that will give after the game end the normed averaged payoff $v_{00}(c_1, c_2; G, R)$, which must be positive or negative for the first or the second player respectively. Such games are typical for the systems or processes, when one of the players is not a person, but, say, is the nature or something inanimate. And the main task, arising in it, is for the person to foresee the schedule of the pure strategies selection by this inanimate one, that will be restated into the corresponding PSST of the inanimate. After this procedure there remains a problem of the decision-making theory — just to find the optimal PSST of the person for applying it further.

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